18th Year of Quality education

Code No. **Series AG-R** CLASS XII **TMG -D/79/89**

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours **Maximum Marks : 100**

PART – A

1. Evaluate : $\int \frac{dx}{\cos(x-a)\cos(x-b)}$.
2. If $4\sin^{-1} x+\cos^{-1} x=\pi$ then find the value of x .
3. There are three mutually exclusive and exhaustive events E_1. E_2 and E_3. The odds are 8:3 against E_1 and 2:5 in favor of E2. Find the odds against E_3.
4. Evaluate : $\int \frac{dx}{x^2(x^4+1)^{3/4}}$.
5. At what points of the ellipse $16x^2 + 9y^2 = 400$, does the ordinates decrease at the same rate at which the abscissa increase ?
6. Find the inverse element of the binary relation $a\otimes b=a+b-4$.
7. Given $\vec{a}.\vec{b}=\left|\vec{a}\times\vec{b}\right|$ find the angle between $\vec{a}$ & $\vec{b}$.
8. If $A=\begin{bmatrix}3 & -4\\ -1 & 2\end{bmatrix}$ find matrix B such that AB = I .
9. The slope of tangent to curve $y=\frac{x-1}{x-2}$ at $x=10$.
10. If $A^2=A$ for $A=\begin{bmatrix}-1 & b\\ -b & 2\end{bmatrix}$, then find the value of b .

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PART – B

11. If $y=\cot^{-1}(\sqrt{\cos x})-\tan^{-1}(\sqrt{\cos x})$ Prove that $\sin y=\tan^2\frac{x}{2}$.

OR

Solve for x : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$.

12. If $y=x^x$ then prove that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$.

13. Suppose a girl throws a die . If she gets a 5 or 6 , she tosses a coin three times and note the number of heads . If she gets a 1 , 2, 3 or 4 , she tosses a coin once and notes whether a heads or tail is obtained . If she obtained exactly one head ;what is the probability that she threw 1 , 2 , 3 or 4 with the die .

14. If $f(x)=\begin{cases}\frac{x-4}{|x-4|}+a & x\langle 4\\ a+b & x=4\\ \frac{x-4}{|x-4|}+b & x\rangle 4\end{cases}$ Determine the values of a and b so that $f(x)$ is continuous at x=4 .

15. If a , b,c is real numbers and $\begin{vmatrix}b+c & c+a & a+b\\ c+a & a+b & b+c\\ a+b & b+c & c+a\end{vmatrix}=0$.Show that either a + b+c = 0 or a = b = c .

16. A plane meets the coordinate axis in A , B , C such that the centroid of triangle ABC is the point (p, q , r) . Prove that the equation of plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$.

17. Evaluate : $\int\frac{\tan\theta+\tan^3\theta}{1+\tan^3\theta}dx$..

OR

Evaluate: $\int_0^{\pi/2}\frac{\cos x}{1+\cos x+\sin x}dx.$

18. Solve the differential equation: $\frac{d^2x}{dy^2}=y\sin^2 y$.

19. Let $N\times N$ be the set of ordered pairs of natural numbers . Also let R be the relation in $N\times N$, defined by $(a,b)R(c,d)\Leftrightarrow ad=bc$ .Show that R is an equivalence relation .

20. Find the shortest distance between the lines whose vector equations are $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2i+3j+4k)$ & $\vec{r}=(2\hat{i}+4\hat{j}+5\hat{k})+\mu(4i+6j+8k)$. Also find the angle between two lines .

OR

Find the S.D. between two skew lines . $\frac{X-8}{3}=\frac{Y+9}{-16}=\frac{10-Z}{-7}$ & $\frac{X-15}{3}=\frac{58-2Y}{-16}=\frac{Z-5}{-5}$. Also find the angle between two lines .

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21. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$ where $\vec{a} = 3i + 2j + 2k$ and $\vec{b} = i + 2j - 2k$.

22. Evaluate : $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$.

PART – C

23. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice . Find the mean or expectation of X .

OR

The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution.

24. A firm manufactures two types of product A and B and sells them at a profit of Rs 5 per unit of type A and Rs. 3 per unit of type B. Each product is processed on two machines M_1 & M_2 .One out of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 , whereas one minute of type B requires one minute of processing time on M_1 and one minute on M_2 . Machine M_1 & M_2 are respectively available for at most 5 hour and 6 hours in a day . Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the graphically.

25. Using integration, find the area of the two parabolas $4y^2 = 9x$ & $3x^2 = 16y$ .Also find thew angle between two curves .

OR

Prove that the curves $y^2 = 4x$ & $x^2 = 4y$ divide the area of square bounded by x = o , x = 4 , y = 4and y = 0 into three equal parts .

26. Water is running into a conical vessel ,15 cm deep and 5 cm in radius , at the rate of 0.1 cm^3 / sec . When the water is 6cm deep, find at what rate is (i) the water level rising? (ii) The water – surface area increasing? (iii) The wetted surface of the vessel increasing?

27. In a hurdle race , a player has to cross 10 hurdles . The probability that he will clear each hurdle is 5 / 6 .What is the probability that he will knock down fewer than 2 hurdles ?

28. Determine the product $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$ and use it to solve the system of equations : $x + 2y + z = 7$; $x + 3z = 11$; $2x - 3y = 1$.

29. A cylinder of greatest volume is inscribed in a cone, show that (i) $R = \frac{2}{3} h \tan\alpha$ (ii) $H = \frac{1}{3} h$ (iii) Volume of the cylinder = $\frac{4}{27}\pi h^3 \tan^2 \alpha$. (iv) r : R = 3 : 2. Where r, h, α are the radius, height and semi – vertical angle of the cone and R, H are the radius and height of the inscribed cylinder.

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